Exercise 52

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(t) = (t^2 - 4)^3, \quad [-2, 3]$$

Solution

Take the derivative of the function.

$$\begin{aligned} f'(t) &= \frac{d}{dt}(t^2 - 4)^3 \\ &= 3(t^2 - 4)^2 \cdot \frac{d}{dt}(t^2 - 4) \\ &= 3(t^2 - 4)^2 \cdot (2t) \\ &= 6t(t^2 - 4)^2 \end{aligned}$$

Set $f'(t) = 0$ and solve for t.

$$6t(t^2 - 4)^2 = 0$$

$$6t(t + 2)^2(t - 2)^2 = 0$$

$$t = \{-2, 0, 2\}$$

$t = -2$ and $t = 0$ and $t = 2$ are within $[-2, 3]$, so evaluate f at these values.

$$f(-2) = [(-2)^2 - 4]^3 = 0$$

$$f(0) = (0^2 - 4)^3 = -64 \qquad \text{(absolute minimum)}$$

$$f(2) = (2^2 - 4)^3 = 0$$

Now evaluate the function at the other endpoint of the interval.

$$f(3) = (3^2 - 4)^3 = 125 \qquad \text{(absolute maximum)}$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[-2, 3]$.

The graph of the function below illustrates these results.